Atmospheric Aerosols – Size Distribution Functions

Atmospheric aerosols are tiny particles dissolved in air. Aerosols at all locations show a large variation in sizes, usually measured in diameters, spanning several orders of magnitude. Some of the particles have sizes of a few nanometer, some up to 100µm in diameter.

Measurement of particle sizes:

Instruments measuring particle diameters use discrete intervals to count particles in certain intervals rather than all particle size ranges and show them graphically. Sometimes a cumulative distribution is given meaning all particles smaller or the same size as the shown one. The last value of a cumulative distribution shows the total number of particles counted over all size ranges.

Different size bins are used to count particles, for examples a size range from 0.01µm to 0.02µm and then a size bin from 0.32µm to 0.64µm. Such different bin sizes make it rather difficult to interpret absolute concentrations. To avoid biases, normalization of the distribution by dividing the concentration with the corresponding size range is performed. The result is a concentration expressed in µm⁻¹cm⁻³. Now the area below the curve is proportional to the number concentration. Often a logarithmic scale is used in order to present the smaller sizes in an easier readable form.

The Number Distribution nₙₙ(Dₚ):

We can calculate aerosol concentrations by integration over the whole size range and making the size intervals infinitesimally small:

First approximation:

\[ N_i = n_i \Delta D_p \]

\( n_i \): aerosol distribution for a size interval i
\( N_i \): absolute aerosol concentration
\( \Delta D_p \): size range

Such arbitrary intervals make the intercomparison rather difficult and in order to avoid complications and maintain the aerosol distribution information smaller and smaller size bins to the limit \( \Delta D_p \rightarrow 0 \) are used. \( \Delta D_p \) becomes \( dD_p \) and we can define the size distribution function as follows:
\( n_N(D_p) \, dD_p = \) the number of particles per cm\(^3\) of air having diameters in the range \( D_p \) to \( D_p + dD_p \).

The units are \( \mu m^{-1} \, cm^{-3} \) and the total number of particles – cm\(^{-3}\) is:

\[
N = \int_0^\infty n_N(D_p) \, dD_p
\]

Usually particle size functions are displayed as the normalized size distribution function \( n_N(D_p) \) by

\[
\bar{n}_N(D_p) = \frac{n_N(D_p)}{N}
\]

\( n_N(D_p) \) being the fraction of the total number of particles per cm\(^3\) having diameters in the range \( D_p \) to \( D_p + dD_p \). Units = \( \mu m^{-1} \).

The normalized size distribution is expressed in the following way with \( dN = n_N(D_p) \, dD_p \) as the number of particles in the size range \( (D_p, D_p + dD_p) \) and \( n_N(D_p) \) is:

\[
n_N(D_p) = \frac{dN}{dD_p}
\]

Both sides represent the same aerosol distribution and the notation of the right side of the equation is usually used to express aerosol sizes.

**Surface Area, Volume and Mass Distributions:**

Sometimes it is more convenient to use surface, volume or mass distributions with respect to particle size as several properties are depending on them.

The aerosol surface area distribution \( n_S(D_p) \) is defined in the following way:

\[
n_S(D_p) \, dD_p = \) corresponding to the surface area of particles per cm\(^3\) of air having diameters in the range \( D_p \) to \( D_p + dD_p \).

All particles in this infinitesimally narrow size range have effectively the same diameter \( D_p \), and each of them has the surface area \( \pi D_p^2 \). Since there are \( n_N(D_p) \, dD_p \) particles in this size range their surface area is \( \pi D_p^2 n_N(D_p) \, dD_p \).
\begin{align*}
n_s(D_p) &= \pi D_p^2 n_N(D_p) \\
\text{in } [\mu m \cdot cm^{-3}] \\
\end{align*}

with the total surface area given by:

\begin{align*}
S &= \pi \int_{D_p}^{\infty} D_p^2 n_N(D_p) \, dD_p = \int_{D_p}^{\infty} n_s(D_p) \, dD_p \\
\text{in } [\mu m^2 \cdot cm^{-3}] \\
\end{align*}

The aerosol volume distribution can be defined in the same way as:

\begin{align*}
n_v(D_p) \, dD_p \quad \text{is corresponding to the volume of particles per cm}^3 \quad \text{of air having} \\
\text{diameters in the range } D_p \text{ to } D_p + dD_p. \\
\end{align*}

Therefore:

\begin{align*}
n_v(D_p) &= \frac{\pi}{6} D_p^3 n_N(D_p) \\
\text{in } [\mu m^2 \cdot cm^{-3}] \\
\end{align*}

and for the total aerosol volume per cm\(^3\) of air, \(V\) is:

\begin{align*}
V &= \frac{\pi}{6} \int_{D_p}^{\infty} D_p^3 n_N(D_p) \, dD_p = \int_{D_p}^{\infty} n_v(D_p) \, dD_p \\
\text{in } [\mu m^3 \cdot cm^{-3}] \\
\end{align*}

For particles with the same density \(\rho_p\) (g/cm\(^3\)) the distribution of particle mass with respect to particles size is given by:

\begin{align*}
n_M(D_p) &= \left(\frac{\rho_p}{10^6}\right) n_v(D_p) = \left(\frac{\rho_p}{10^6}\right) \left(\frac{\pi}{6}\right) D_p^3 n_N(D_p) \\
\text{in } [\mu g \cdot \mu m^{-1} \cdot cm^{-3}] \\
\end{align*}
The size distributions are often in a more convenient logarithmic way as \( \ln D_p \) or \( \log D_p \) expressed.

\[ n^e_N(\ln D_p) d\ln D_p \]

is corresponding to the particles per \( cm^3 \) of air in the size range \( \ln D_p \) to \( \ln D_p + d\ln D_p \).

with the total number distribution \( N \) as:

\[
N = \int_{\ln 0}^{\ln \infty} n^e_N(\ln D_p) d\ln D_p
\]

\[ cm^{-3} \]

The same we can express for the surface area and volume distributions:

Surface area

\[
S = \int_{-\infty}^{\infty} D_p^2 n^e_N(\ln D_p) d\ln D_p = \int_{-\infty}^{\infty} n^e_S(\ln D_p) d\ln D_p
\]

Volume

\[
V = \int_{-\infty}^{\infty} D_p^3 n^e_N(\ln D_p) d\ln D_p = \int_{-\infty}^{\infty} n^e_V(\ln D_p) d\ln D_p
\]
Types of aerosols

Urban aerosol:

Urban aerosols are mixtures of particles with different origin: primary particulate emissions from industries, transportation, power generation, natural sources and secondary particles formed by gas-to-particle conversion. The number distribution has its peak below 0.1\(\mu\)m, whereas the surface area distribution has its peak between 0.1 and 0.5\(\mu\)m. This feature inhibits that the mass transfer of material from the gas phase during gas-to-particle conversion occurs preferably on them. The volume distribution is usually dictated by a bimodal or even trimodal function, with two peaks in the submicron range – nuclei and accumulation mode particles- and another one in the coarse particle range. Between 1.0 and 3 \(\mu\)m exists usually a gap. The sources of the two size modes are different: coarse particles are generated by mechanical processes and consist of soil dust, fly ash, tire wear particles etc., whereas nuclei and accumulation mode particles contain primary particles from combustion sources and secondary aerosol material such as sulfate, nitrate, ammonia, secondary organics, formed by chemical reactions resulting in gas-to-particle conversion.

Marine Aerosols:

In remote marine regions without any anthropogenic sources and low transport of continental aerosol, particles are mostly of marine origin. They show fairly low number concentrations of 100 –300cm-1 and a typically tri-modal size distribution peaking in nuclei, accumulation and coarse mode.

Rural continental aerosols:

Aerosols in rural areas are mainly of natural origin with moderate influences of anthropogenic sources. The mass or volume concentrations are dominated by coarse particles with size of about 7\(\mu\)m, while the number distribution shows two peaks at about 0.02\(\mu\)m and 0.08\(\mu\)m respectively.

Remote continental Aerosol:

Remote continental aerosols show a stronger bimodal number concentration as rural ones. Particles smaller than 2.5\(\mu\)m in diameter represent about 40-80% of the total aerosol mass and consist mainly of sulfate, ammonium and organics originated from dust, pollen and plant waxes.

Free Tropospheric Aerosols:
Free tropospheric aerosols are usually taken on high alpine stations or by aircraft. They show a large peak in the mass/volume distribution around 1µm and mean diameters in the number distribution of 0.01 and 0.25µm. Such free tropospheric spectra indicate that smaller particles were scavenged by precipitation or deposited already.

*Polar Aerosols:*

Polar aerosols are characterized by very low particle concentrations. During winter and spring the Arctic aerosol has been found to be influenced significantly by anthropogenic sources – Arctic Haze- rising the number concentrations of this aerosol type significantly. Polar aerosols are aged and often contain carbonaceous material from midlatitude pollution sources, sea salt, mineral dust from arid regions.

*Desert Aerosols:*

Desert aerosols experience typically large volume and surface distributions peaking in the coarse mode. The number distribution often is broad and has three overlapping modes at 0.01µm, 0.05µm and 20µm.